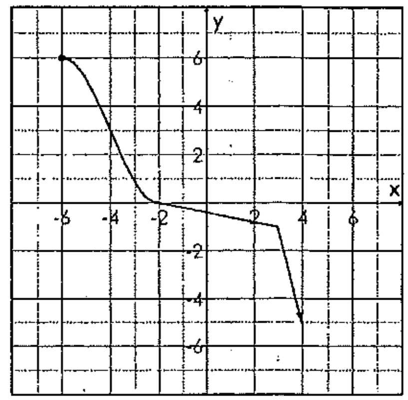
Math 150
Final Review

This review, like the final, is based on the learning outcomes passed out at the start of the semester. While you are expected to have and use a graphing calculator during the final, it is also expected that you will know how to solve most problems by hand and will need to show work to that effect. The chapter reviews after each chapter in our textbook can also serve as helpful review.

Part A :: Functions



1. Consider the relation graphed to the right.
   - **a.** Explain why the relation is a function.
   - **b.** Is the function even, odd, or neither? Explain.
   - **c.** Is the function one-to-one? Explain.
   - **d.** If possible, sketch the graph of the inverse function. If not possible, explain.
   - **e.** State the domain of the function.
   - **f.** State the range of the function.

2. Consider the functions $f(x) = \dfrac{x^2 + 3}{x}$ and $g(x) = \dfrac{x + 1}{7}$.
   - **a.** Determine whether f is even, odd, or neither algebraically.
   - **b.** Find and simplify the difference quotient of f.
   - **c.** Determine whether g is one-to-one algebraically.
   - **d.** If possible, find $g^{-1}(x)$. If not possible, explain.

3. Let $g(x) = \dfrac{x^4 + 2x^2 - 3}{x^2 - 9}$ and $h(x) = \sqrt{4 - 3x}$. Determine $(g \circ h)(-3)$.

4. Let $f(x) = \sin(x + \pi/2)$ and $g(x) = \log_3(4x+13)$. Determine $(g \circ f)(\pi)$.

5. List the potential zeros of the function $f(x) = 2x^5 - x^3 + 2x^2 + 12$. Do not attempt to find the actual zeros.

6. Use the Intermediate Value Theorem to show that $y = 2x^3 + 6x^2 - 8x + 2$ has a zero in the interval [-5, -4].

7. Use the remainder theorem to determine the remainder when $f(x) = 2x^{20} - 8x^{10} + x - 2$ is divided by $x - 1$.

8. Write as a single logarithm: $4\ln x - [\ln(x+1) + 2\ln(x - 1)]$

9. Determine $\log_7 15$ to five decimal places.

10. Without using a calculator, determine the exact value of each of the following trigonometric functions:

a. $\cos 540°$

b. $\sin^{-1}\left(\sin\left(\frac{9\pi}{8}\right)\right)$

c. $\cot\left(\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$

11. Given that $\sec\theta = 4$ and $\tan\theta < 0$, determine the value of the six trigonometric functions.

12. Write a sine function with an amplitude of 5, a period of 3π, and a phase shift of $-\frac{2}{3}$.

Part B :: Equations & Inequalities

13. Solve: $2x^2 + 8x < 10$

14. Solve: $x(x-1)(7-x) \geq 0$

15. Solve: $\frac{2x-6}{1-x} < 2$

16. Solve: $\log_2(x+7) + \log_2(x+8) = 1$

17. Solve: $2^{x+1} = 5^{1-2x}$
Write your answer **a.** as a log in base 5 and **b.** rounded to the nearest thousandth.

18. Solve: $3^{2y} + 3^y - 2 = 0$

19. Solve: $\log(x-3) = -2$

20. Solve: $4^x - 16^{2x+1} = 0$

21. Solve: $\cos^2\theta - 3\cos\theta = -2$ on $[0, 2\pi)$

22. Solve: $\sqrt{3}\sin\varphi - \cos\varphi = 1$ on $[0, 2\pi)$

Part C :: Graphing & Applications

23. Write $f(x) = x^2 + 2x - 3$ in vertex form. Then sketch the graph of the parabola using the axis of symmetry, intercepts, and the vertex.

24. Sketch the graph of $g(x) = 2f(x-3) + 1$ using transformations, where $f(x) = e^x$.

25. Sketch the graph of $h(x) = (x-4)(x+2)^2(x-2)$ using the steps on page 179.

26. Sketch the graph of $q(x) = \dfrac{x^2 + x - 6}{x^2 - x - 6}$ using the steps on pages 201-202.

27. Sketch the graph of $T(x) = 3\sin(2x - \pi/3)$ over at least two periods.

28. Sketch the graph of $f(x) = \cot(x + \pi/2)$ over at least two periods.

29. Analyze and sketch $\dfrac{(x-3)^2}{4} - \dfrac{y^2}{36} = 1$

30. Analyze and sketch $2x^2 + 3y^2 - 8x + 12y - 4 = 0$

31. Establish each of the following trig functions:

    a. $\sin\theta \tan\theta + \cos\theta = \sec\theta$

    b. $\dfrac{1 - 2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$

32. David has 400 yards of fencing and wishes to enclose a rectangular area.
    a. Express the area A as a function of the width w of the rectangle.
    b. For what value of w is the area largest?
    c. What is the maximum area?

33. The half-life of a radioactive element is 1,690 years. If 10 grams are present now, how much will be present in 50 years? Round to the nearest hundredth of a gram.

34. Ted has $1,000 to invest at 8% annually. How long will it take for Ted to have $1,500 if the interest is **a.** simple **b.** compounded monthly **c.** compounded continuously? Round your answers to two decimal places.

Answer Key

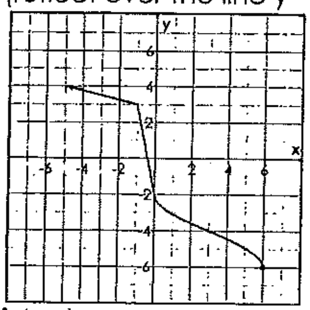
PART A

1. **a.** it passes the vertical line test

b. neither; it is symmetric about neither the y-axis (even) nor the origin (odd)

c. yes; it passes the horizontal line test

d. (reflect over the line $y = x$)



e. $[-6, \infty)$

f. $(-\infty, 6]$

2. **a.** odd

b. $\dfrac{x^2 + xh - 3}{x(x + h)}$

c. it is one-to-one

d. $g^{-1}(x) = 7x - 1$

3. 48

4. 2

5. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm 1\frac{1}{2}$

6. $f(-5) = -58$ and $f(4) = 2$. Because $f(-5) < 0 < f(4)$, there is a zero on $[-5, 4]$.

7. $f(1) = -7$

8. $\ln\left(\dfrac{x^4}{(x + 1)(x - 1)^2}\right)$

9. 1.39166

10. **a.** -1

b. $-\dfrac{\pi}{8}$

c. $-\dfrac{\sqrt{2}}{2}$

11. $\sin\theta = -\dfrac{\sqrt{15}}{4}$ $\quad\csc\theta = -\dfrac{4\sqrt{15}}{15}$

$\cos\theta = \dfrac{1}{4}$ $\quad\sec\theta = 4$

$\tan\theta = -\sqrt{15}$ $\quad\cot\theta = -\dfrac{\sqrt{15}}{15}$

12. $f(x) = 5\sin\left(\dfrac{2}{3}x + \dfrac{4}{9}\right)$

PART B

13. $(-5, 1)$

14. $(-\infty, 0] \cup [1, 7]$

15. $(-\infty, 1) \cup (2, \infty)$

16. $\{-6\}$

17. **a.** $\dfrac{1 - \log_5 2}{\log_5 2 + 2}$

b. 0.2342

18. $y = 0$

19. 3.01

20. $-\frac{2}{3}$

21. 0

22. $\left\{\dfrac{\pi}{3}, \pi\right\}$

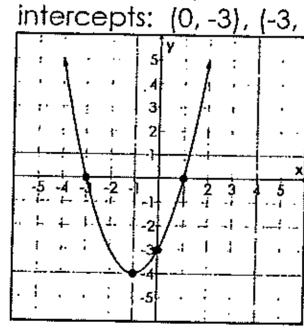
PART C

23. $f(x) = (x + 1)^2 - 4$

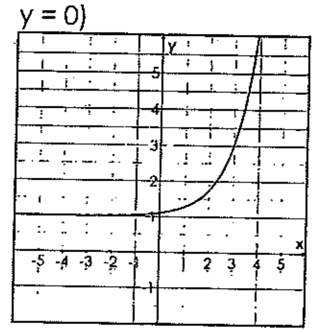
axis of symmetry: $x = -1$

vertex: $(-1, -4)$

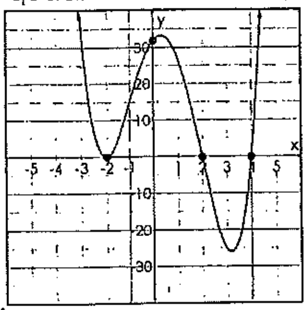
intercepts: $(0, -3)$, $(-3, 0)$, $(1, 0)$



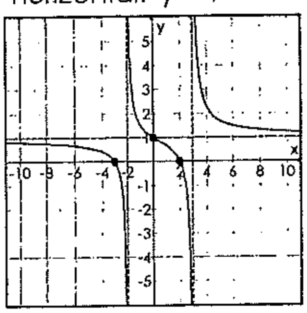
24. shift $f(x) = e^x$ right 3, stretch vertically by 2, and then shift up one. The horizontal asymptote also moves up to $y = 1$ (from $y = 0$)



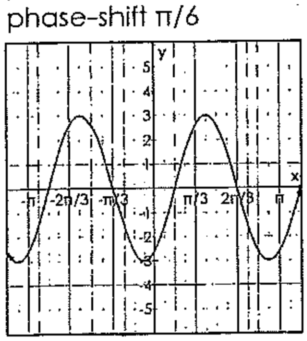
25. * Behaves like $y = x^4$
* y-int: (0, 32)
  x-int: (4, 0), (-2, 0), (2, 0)
* -2 has multiplicity 2, so it turns around; the others have mult. 1, so they cross
* max turning pts is 3
* linear with pos. slope near x = 4
  linear with neg slope near x = 2
  quadratic near x = -2, open up



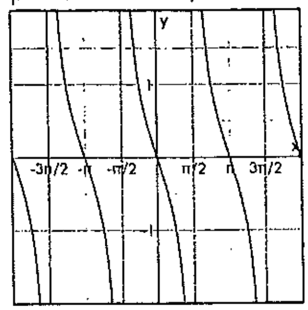
26. * $q(x) = \dfrac{(x+3)(x-2)}{(x-3)(x+2)}$
* domain: { x | x is not 3, -2 }
* q is in lowest terms
* x-ints: (-3, 0) and (2, 0)
* vert. asym: x = 3, x = -2
* horizontal: y = 1



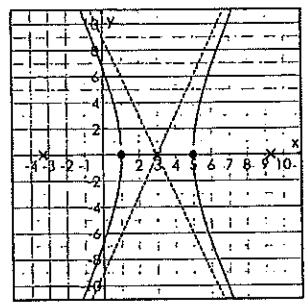
27. amplitude: 3
period: π
phase-shift π/6



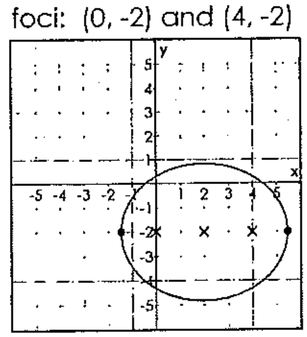
28. amplitude: 1
period: π
phase-shift: π/2



29. Hyperbola (x-type)
center (3, 0)
vertices (1, 0) and (5, 0)
foci $(3 \pm 2\sqrt{10}, 0)$
asymptotes: $y = \pm 3(x - 3)$



30. $\dfrac{(x-2)^2}{12} + \dfrac{(y+2)^2}{8} = 1$
Ellipse
center (2, -2)
major axis: parallel to x-axis
vertices $(2 \pm 2\sqrt{3}, -2)$
foci: (0, -2) and (4, -2)



31. a.

$$\sin\theta\tan\theta + \cos\theta$$
$$= \sin\theta\frac{\sin\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$$
$$= \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta}$$
$$= \sec\theta$$

#

b.

$$\tan\theta - \cot\theta$$
$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta}{\cos\theta\sin\theta} - \frac{\cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{(1 - \cos^2\theta) - \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{1 - 2\cos^2\theta}{\cos\theta\sin\theta}$$

#

32. **a.** A = w(200-w)
b. w = 100 yards
c. 10,000 sq yards

33. 9.797 grams

34. **a.** 6.25 years
b. 5.09 years
c. 5.07 years