Math 151
Review for the Final

Here you have some questions that may help you to be better prepared for the final exam. You should also review the homework problems in WebAssign and the previous tests.

PART A

Limits Find each limit **algebraically**.

1. $\lim\limits_{t \to 2} \dfrac{t^2 + 3t - 10}{t^2 + t - 6}$

2. $\lim\limits_{\theta \to \pi/2} \sqrt{\sin\theta + 3}$

3. $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$

4. $\lim\limits_{x \to 1} \dfrac{\sqrt{x+2} - \sqrt{3}}{x - 1}$

5. $\lim\limits_{x \to -\infty} x^3 + 5x^4$

6. $\lim\limits_{x \to \infty} \dfrac{3x^3 + 4x^2 + 1}{(x^2 + 1)(x - 2)}$

Derivatives Use the differentiation rules to:

7. Differentiate: $f(x) = \cos^2(3x+1)$.
8. Find the derivative of $g(x) = \left(x - \sqrt{x^2 + 1}\right)^3$.
9. Determine $\dfrac{d}{dx}\left[\, 2x(\sin x)(\sec 2x) \,\right]$.
10. Find y' when $y = \dfrac{2x^3 - 5}{\sqrt{x} + 1}$.
11. Find the derivative of $h(x) = \dfrac{\sin x + 1}{\cot x - 1}$.
12. Differentiate: $F(x) = 3x^3 - 2x^2 + 5x^5$
13. Find the second derivative of $y = \cos t - t \sin t$.
14. Find the second derivative of $y = (x + 1)^2(x - 1)$.

Integrals Find each integral using the properties and techniques discussed in class.

15. $\int_{\pi/4}^{3\pi/4} (\csc^2 x + 15x^4)\, dx$ **16.** $\int_1^3 \sqrt{\frac{9}{x}}\, dx$

17. $\int_0^2 \sin \pi x \cos^2 \pi x\, dx$ **18.** $\int_{-4}^{-2} 3x^2(x^3 + 2)^4\, dx$

19. $\int -5 \sec\theta \tan\theta\, d\theta$ **20.** $\int (2x^5 - 4x^3 + x - 2)\, dx$

21. $\int 6x \cos(3x^2)\, dx$ **22.** $\int 18x\sqrt{x^2 - 5}\, dx$

Mandatory *The following problems are especially important, and for that reason they has been labeled "mandatory problems"*

23. Use the **definition** of the derivative to determine $g'(-1)$ when $g(x) = x^2 + 2x - 4$.

24. Using the definition of the definite integral, and taking x_i^* to be the right end-points, write $\int_0^4 (2x^2 + 3)\, dx$ as a limit of Riemann sums You do not need to evaluate your limit.

25. Find an equation of the tangent line to the curve $f(x) = \sec x + \tan x$ at the point $\left(\frac{\pi}{6}, \sqrt{3}\right)$.

26. Find y' implicitly when $2x^3 + 4xy^{-1} = 5x + 1$.

27. Determine $\frac{d}{dx}\int_1^{x^2+4} \frac{t+3}{t^2+9}\, dt$.

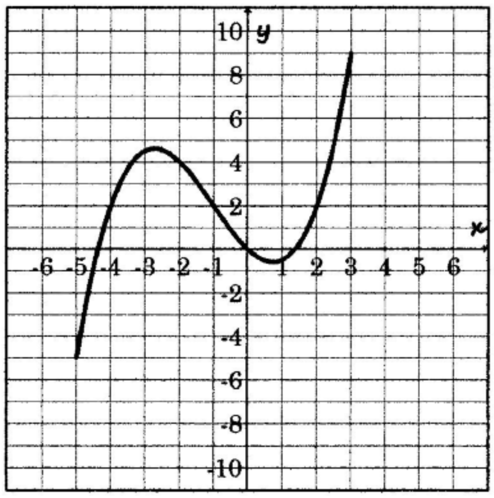
PART B

1. Let $g(x) = \begin{cases} x^2 - 3 & \text{if } -3 < x \leq 2 \\ 3 - x & \text{if } 2 < x < 6 \end{cases}$

 a. Show that g is continuous on $(-3, 6)$.

 b. Show that g is not differentiable at $x = 2$.

2. Estimate $\int_{-5}^{3} f(x)dx$ using M_4 where $f(x)$ is the function graphed to the right.



3. On Planet Ω, the Space Critters are trying to maximize the grazing area for their Space Sheep. If they have 3,600 space-feet of fencing, and they need to put fence around three sides of a rectangular plot of land (the fourth is alongside a Space River and doesn't need a fence), determine the maximum area they can fence-off. Round your answer to the nearest hundredth if necessary.

4. a. Find the area under $y = 2x^3 + 1$ on $[-1, 2]$.

 b. Determine $\int_{1}^{2} f(x)dx + \int_{2}^{0} f(x)dx$ given the following:

 $\int_{0}^{1} f(x)dx = 2 \qquad \int_{1}^{3} f(x)dx = 5 \quad \text{and} \quad \int_{3}^{2} f(x)dx = -2$

5. Samson wanders through a village for 3 hours. His velocity can be modeled by the function $v(t) = 3t^2 - 10t + 3$ in miles per hour.

 a. At the end of the 3 hours, how far away is he from his starting position?

 b. What is the total distance he travels during these three hours?

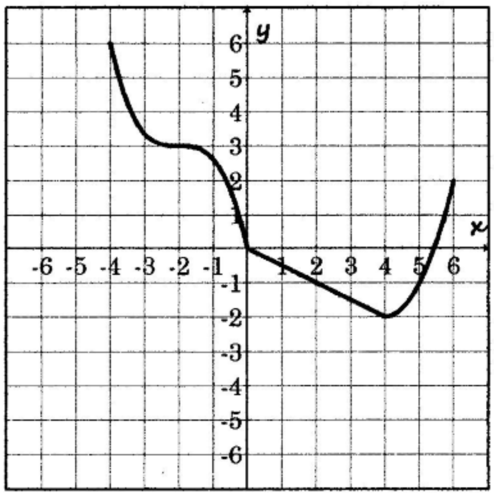
6. Suppose the velocity of a particle is given by $v(t) = 8t^3 - 2\sin t$, in m/s.

   a. Find the acceleration at $t = 1$. Round your answer to the nearest hundredth.

   b. If $s(0) = 3$, find a function for the particle's displacement.

7. A snail is walking west toward a twig at 5 cm/hr and a turtle is walking north toward the same twig at 7 cm/hr. At 3:00, the snail is 3 m from the twig and the turtle is 4 m from the twig. At what rate are the turtle and snail approaching each other at 3:00?

8. Estimate $\int_{-4}^{6} f(x)dx$ using R_5 where $f(x)$ is the function graphed to the right.



ANSWER KEY - Part A

LIMITS

1. $^7/_5$
2. 2
3. 1
4. $\frac{\sqrt{3}}{6}$
5. ∞
6. 3

DERIVATIVES

Note: The form of your answer may differ from the form given.

7. $-6\cos(3x+1)\sin(3x+1)$
8. $3\left(x-\sqrt{x^2+1}\right)^2\left(1-\frac{x}{\sqrt{x^2+1}}\right)$
9. $4x\sin x\sec 2x\tan 2x + 2\sin x\sec 2x + 2x\cos x\sec 2x$
10. $\frac{10x^3\sqrt{x} + 12x^3 + 5\sqrt{x}}{2x(\sqrt{x}+1)^2}$
11. $\frac{\cos x\cot x - \cos x + \csc x + \csc^2 x}{\csc^2 x - 2\cot x}$
12. $9x^2 - 4x + 25x^4$
13. $y'' = -3\cos t + t\sin t$
14. $y'' = 6x + 2$

INTEGRALS

15. $2 + \frac{363\pi^5}{512}$
16. $6(\sqrt{3} - 1)$
17. 0
18. 183,225,011.2
19. $-5\sec\theta + C$
20. $^1/_3x^6 - x^4 + {^1/_2}x^2 - 2x + C$
21. $\sin(3x^2) + C$
22. $6(x^2-5)^{3/2} + C$

MANDATORY

23. $$\lim_{x\to -1} \frac{g(x) - g(-1)}{x-(-1)} = \lim_{x\to -1} \frac{(x^2+2x-4)-(1-2-4)}{x+1}$$

$$= \lim_{x\to -1} \frac{x^2+2x-4+5}{x+1}$$

$$= \lim_{x\to -1} \frac{x^2+2x+1}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)^2}{x+1}$$

$$= \lim_{x\to -1} x+1$$

$$= -1+1$$

$$= \mathbf{0}$$

24. $$\lim_{x\to\infty} \sum_{i=1}^{n} \left(\frac{32i^2}{n^2}+3\right)\frac{4}{n}$$

25. $y = 2\left(x-\frac{\pi}{6}\right) + \sqrt{3}$

26. $y' = \frac{6x^2y^2+4y-5y^2}{4x}$

27. $\frac{2x^3+14x}{x^4+8x^2+25}$

PART B

1. Since $g(x) = \begin{cases} x^2 - 3 & \text{if } -3 < x \leq 2 \\ 3 - x & \text{if } 2 < x < 6 \end{cases}$

a. g is continuous on (–3, 2) U (2, 6) because it is a polynomial there, but we need to show continuity at $x = 2$.

$g(2) = 1$ and $\lim_{x \to 2^-} g(x) = 1 = \lim_{x \to 2^+} g(x)$

Therefore, $\lim_{x \to 2} g(x) = g(2)$, so g is also continuous at $x = 2$.

b. g has a cusp point at $x = 2$ (the derivative from the left at 2 is 4, but the derivative from the right is -1), so g is not differentiable at $x = 2$.

2. $\int_{-5}^{3} f(x)dx \approx M_4 = 2[f(-4) + f(-2) + f(0) + f(2)] = 2[2 + 4 + 0 + 2] = 16$

3. $P = 3600\, ft$ and $2x + y = 3600$

We want to max: $A = xy = x(3600 - 2x) = -2x^2 + 3600x$

Then, $A'(x) = -4x + 3600 = 0 \quad \Rightarrow x = 900$

Therefore, the maximum area they can fence-off is $A = 1{,}620{,}000\, ft^2$

4. **a.** To find the area under $y = 2x^3 + 1$ on [–1, 2], we evaluate the integral

$\int_{-1}^{2} (2x^3 + 1)\, dx = \frac{1}{2}x^4 + x\Big]_{-1}^{2} = 10\frac{1}{2}$

b. Determine $\int_1^2 f(x)dx + \int_2^0 f(x)dx$ given the following:

$\int_0^1 f(x)dx = 2 \qquad \int_1^3 f(x)dx = 5 \quad$ and $\quad \int_3^2 f(x)dx = -2$

We have: $\int_1^2 f(x)dx + \int_2^0 f(x)dx = \int_1^2 f(x)dx - \int_0^2 f(x)dx = -\int_0^1 f(x)dx = -2$

5. Since $v(t) = 3t^2 - 10t + 3$ we have:

a. $s(t) = t^3 - 5t^2 + 3t$ and the displacement is $s(3) - S(0) = -9\, mi$. This means that at the end of the 3 hours, Samson will be 9 miles to the left away from his starting position.

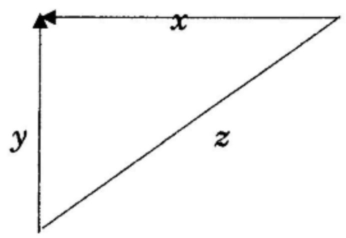
b. He will stop when $v(t) = (3t - 1)(t - 3) = 0 \;\rightarrow\; t = \frac{1}{3}, t = 3$; then, the total distance he travels during these three hours is $\int_0^{\frac{1}{3}} v(t)dt - \int_{\frac{1}{3}}^{3} v(t)dt = \frac{269}{27} = 9\frac{26}{27}\, miles$

6. Since the velocity of a particle is given by $v(t) = 8t^3 - 2\sin t$ m/s then:

a. The acceleration at $t = 1$ is $a(t) = v'(t) = 24t^2 - 2cost \ \rightarrow a(1) = 22.92\ m/s^2$.

b. $s(t) = \int v(t)\,dt = 2t^4 + 2cost + C$ and $s(0) = 3$, then $s(t) = 2t^4 + 2cost + 1$ is the function for the particle's displacement.

7.



$x^2 + y^2 = z^2 \ \rightarrow\ 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x = 300\ cm, y = 400\ cm\ and\ z = 500cm$

Then, $2(300)(-5) + 2(400)(-7) = 2(500)\frac{dz}{dt}\ \rightarrow$

$$\frac{dz}{dt} = -8.6\ cm/h$$

8. $\int_{-4}^{6} f(x)dx \approx R_5 = 2[f(-2) + f(0) + f(2) + f(4) + f(6)] = 2[3 + 0 + (-1) + (-2) + 2] = 4$