

# Summary of Transformations

Graph	Draw the graph of $f(x)$ and:	Changes in $f(x)$
<b>Vertical shift</b> $y = f(x) + c$ $y = f(x) - c$	Raise the graph of $f(x)$ by $c$ units -add $c$ to $y$ coordinate  Lower the graph of $f(x)$ by $c$ units -subtract $c$ from $y$ coordinate	<p>The graph illustrates vertical shifts of the parabola <math>y = x^2</math>. The blue curve represents the original function <math>x^2</math>. The red curve, labeled <math>x^2 + 5</math>, is shifted 5 units upwards. The green curve, labeled <math>x^2 - 5</math>, is shifted 5 units downwards.</p>
<b>Horizontal shift</b> $y = f(x + c)$ $y = f(x - c)$	Shift the graph $f(x)$ to the left $c$ units -subtract $c$ from $x$ coordinate  Shift the graph $f(x)$ to the right $c$ units -add $c$ to $x$ coordinate	<p>The graph illustrates horizontal shifts of the parabola <math>y = x^2</math>. The blue curve represents the original function <math>x^2</math>. The red curve, labeled <math>(x+3)^2</math>, is shifted 3 units to the left. The green curve, labeled <math>(x-3)^2</math>, is shifted 3 units to the right.</p>
<b>Reflection about the x-axis</b> $y = -f(x)$	Reflect the graph of $f(x)$ about the x-axis -multiply each $y$ coordinate by $-1$	<p>The graph illustrates reflection about the x-axis. The blue curve represents the original function <math>x^2</math>. The red curve, labeled <math>-x^2</math>, is a reflection of the blue curve across the x-axis.</p>
<b>Reflection about the y-axis</b> $y = f(-x)$	Reflect the graph of $f(x)$ about the y-axis -multiply each $x$ coordinate by $-1$	<p>The graph illustrates reflection about the y-axis. The blue curve represents the original function <math>x^3</math>. The red curve, labeled <math>(-x)^3</math>, is a reflection of the blue curve across the y-axis.</p>
<b>Vertical stretching and compression</b> $y = cf(x), c > 1$ $y = cf(x), 0 < c < 1$	Vertically stretching the graph of $f(x)$ ( $c > 1$ )  Vertically compressing the graph of $f(x)$ ( $0 < c < 1$ )  -multiply each $y$ coordinate by $c$	<p>The graph illustrates vertical stretching and compression of the parabola <math>y = x^2</math>. The blue curve represents the original function <math>x^2</math>. The red curve, labeled <math>2x^2</math>, is stretched vertically by a factor of 2. The green curve, labeled <math>\frac{1}{2}x^2</math>, is compressed vertically by a factor of 2.</p>
<b>Horizontal stretching and compression</b> $y = f(cx), c > 1$ $y = f(cx), 0 < c < 1$	Horizontally compressing the graph of $f(x)$ ( $c > 1$ )  Horizontally stretching the graph of $f(x)$ ( $0 < c < 1$ )  -divide each $x$ coordinate by $c$	<p>The graph illustrates horizontal stretching and compression of the cubic function <math>y = x^3</math>. The blue curve represents the original function <math>x^3</math>. The red curve, labeled <math>(2x)^3</math>, is horizontally compressed by a factor of 2. The green curve, labeled <math>(\frac{1}{2}x)^3</math>, is horizontally stretched by a factor of 2.</p>
$y = \frac{1}{f(x)}$	Take the reciprocal of each $y$ coordinate of $f(x)$	
<b>Order of operations for transformations:</b> 1) horizontal shifts 2) stretches/compressions 3) reflections 4) vertical shifts		